PREPARED BY: Dearth

DATE: 17 June 2014

TITLE: Beam/Pendulum SDOF

CHANGE LETTER(S):

Applied Analysis & Technology

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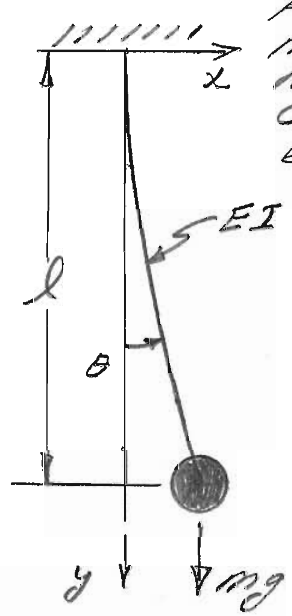

First order approximation for 1st natural (resonant) frequency of massless cantilever beam with concentrated mass at the end. The beam is treated as a spring with spring rate $K = 3EI/l^3$.

Using Hamilton's Principal

$$\delta\int_{t_0}^{t_1}(T+W)\,dt - \frac{\partial T}{\partial \dot q_i}\delta q_i\Big|_{t_0}^{t_1} = 0$$

Kinetic Energy $T = 1/2\, mv^2 = \frac{1}{2} m(\dot x^2 + \dot y^2)$

Work Force x Distance $W = mg(y) - Kx(x)$

Generalized Coordinates $x = l\sin\theta$ $y = l\cos\theta$

$\dot x = l\cos\theta$ $\dot y = -l\sin\theta$

substituting and note $(\cos^2\theta + \sin^2\theta) = 1$

$$\delta\int_{t_0}^{t_1}\left[1/2\, ml^2\dot\theta^2 + (mg)\,l\cos\theta - \left(\frac{3EI}{l^3}\,l\sin\theta\right)l\sin\theta\right]dt - ml^2\dot\theta\,\delta\theta\Big|_{t_0}^{t_1} = 0$$

Taking the variation of Displacements while holding Forces constant.

$$\int_{t_0}^{t_1}\left[ml^2\underbrace{\dot\theta}_{u}\,\underbrace{\delta\dot\theta}_{dv} - (mg)\,l\sin\theta\,\delta\theta - \left(\frac{3EI}{l^3}\,l\sin\theta\right)l\cos\theta\,\delta\theta\right]dt - ml^2\dot\theta\,\delta\theta\Big|_{t_0}^{t_1} = 0$$

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Integrating by parts on variations of derivatives w.r.t. $\int u\,dv = uv - \int v\,du$

and assume small angle approximations $\sin\theta \doteq \tan\theta \doteq \theta$ radians $\cos\theta \doteq 1$

$$\cancel{m\ell^2\dot{\theta}\,\delta\theta}\Big|_{t_0}^{t_1} + \int_{t_0}^{t_1}\left(-m\ell^2\ddot{\theta} - (mg)\ell\theta - \frac{3EI}{\ell}\theta\right)\delta\theta\,dt$$

$$\cancel{-m\ell^2\dot{\theta}\,\delta\theta}\Big|_{t_0}^{t_1} = 0$$

For the integral to vanish, the integrand must also vanish.

$$-m\ell^2\ddot{\theta} - \left(mg\ell + \frac{3EI}{\ell}\right)\theta \overset{must}{=} 0$$

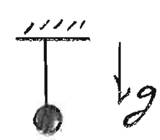
Assume SHM let $\theta = \theta_0 e^{i\omega t}$ $\ddot{\theta} = -\omega^2\theta$

$$m\ell^2\omega^2\theta - \left(mg\ell + \frac{3EI}{\ell}\right)\theta = 0$$

Then $\omega = \sqrt{\frac{3EI}{m\ell^3} + g/\ell}$ rad/sec

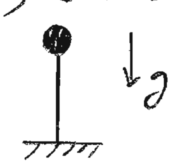
In terms of cycles/sec, Hz

$$\omega = \frac{1}{2\pi}\sqrt{\frac{3EI}{m\ell^3} + g/\ell} \text{ Hz}$$ <u>Down</u>



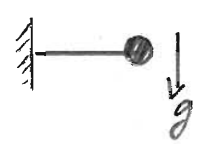
Similarly when the beam (pendulum) is inverted up...

$$\omega = \frac{1}{2\pi}\sqrt{\frac{3EI}{m\ell^3} - g/\ell} \text{ Hz}$$ <u>up</u>



When the beam is Horizontal

$$\omega = \frac{1}{2\pi}\sqrt{\frac{3EI}{m\ell^3}} \text{ Hz}$$



Arithmetic for sample Cantilever Beam

$E := 10.5 \times 10^{6}$ psi $\qquad \nu := 0.3$

$W_1 := 1$ lbs $\qquad g_1 := 386.4 \dfrac{\text{in}}{\text{sec}^2} \qquad m_1 := \dfrac{W_1}{g_1} = 2.588 \times 10^{-3}$ lb $- \dfrac{\text{sec}^2}{\text{in}}$

Beam Dimensions

$b := 1.$ inch $\qquad h := .06$ inch $\qquad L_1 := 18$ inch

$$I := b \cdot \frac{h^3}{12} = 1.8 \times 10^{-5} \quad \text{in}^4$$

Horizontal Resonance

$$\omega_0 := \frac{1}{2\pi} \sqrt{\frac{(3 \cdot E \cdot I)}{\left(L_1^{\,3} \cdot m_1\right)}} = 0.975487 \text{ Hz}$$

Vertical Up Resonance

$$\omega_{up} := \frac{1}{2\pi} \sqrt{\frac{(3 \cdot E \cdot I)}{\left(L_1^{\,3} m_1\right)} - \frac{g_1}{L_1}} = 0.638606 \text{ Hz}$$

Vertical Down Resonance

$$\omega_{dwn} := \frac{1}{2\pi} \sqrt{\frac{(3 \cdot E \cdot I)}{\left(L_1^{\,3} m_1\right)} + \frac{g_1}{L_1}} = 1.222838 \text{ Hz}$$

Critical Buckling Load, P_{cr}, for Vertical Cantilever

$$P_{cr} := \left(\frac{\pi^2}{4}\right) \cdot \frac{(E \cdot I)}{L_1^{\,2}} = 1.439 \quad \text{lbs}_f$$